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Technical paper

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FATIGUE ANALYSIS OF MATERIALS AND METHODS FOR CYCLE COUNTING UNDER ARBITRARY LOAD HISTORIES

ABSTRACT: This paper presents a comparative analysis of cycle counting methods used in fatigue analysis of materials subjected to variable amplitude loading. Several widely used methods, including peak counting, range counting, reservoir method, rainflow counting, and level crossing method, are applied to a representative load history. The obtained results are compared in terms of accuracy, physical relevance, and applicability in engineering practice. The rainflow counting method is identified as the most reliable approach due to its ability to accurately represent load cycles corresponding to material hysteresis behavior.

KEYWORDS: fatigue analysis, cycle counting, rainflow method, variable amplitude loading, structural engineering

ANALIZA ZAMORA MATERIJALA I METODE ODREĐIVANJA BROJA CIKLUSA PRI PROIZVOLJNOJ ISTORIJI OPTEREĆENJA

REZIME: U radu je prikazana komparativna analiza metoda za određivanje broja ciklusa koje se koriste u analizi zamora materijala izloženih opterećenju promenljive amplitude. Više široko primenjivanih metoda, uključujući peak

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counting metodu, range counting metodu, metodu rezervoara, metodu kišnog toka i metodu prelaska nivoa, primenjeno je na reprezentativnu istoriju opterećenja. Dobijeni rezultati su upoređeni sa aspekta tačnosti, fizičke opravdanosti i primenljivosti u inženjerskoj praksi. Metoda kišnog toka izdvojena je kao najpouzdaniji pristup zbog svoje sposobnosti da verno predstavi cikluse opterećenja koji odgovaraju histerezisnom ponašanju materijala.

KLJUČNE REČI: zamor materijala, brojanje ciklusa, metoda kišnog toka, opterećenje promenljive amplitude, građevinsko inženjerstvo

INTRODUCTION

Fatigue in materials can be defined as a microscopic phenomenon that manifests through the initiation and propagation of damage within a structural element or an entire structure. This degradation process occurs even under constant stress levels but is particularly pronounced under cyclic loading conditions.

Fatigue is one of the most common failure mechanisms in engineering structures subjected to repeated loading. Numerous civil engineering structures, including bridges, wind turbines, offshore structures and cranes, experience millions of loading cycles during their service life. Consequently, fatigue assessment has become an indispensable part of structural design and service-life prediction, as comprehensively discussed by Schütz, W (1996).

Besides civil engineering, fatigue plays an equally important role in the automotive industry, where structural components are continuously exposed to complex variable-amplitude loading caused by road irregularities, braking, acceleration, cornering, engine vibrations and thermal loading. The growing demand for lightweight structures, improved fuel efficiency and vehicle electrification has further increased the need for accurate fatigue-life prediction methods capable of analysing realistic service load histories. Modern durability assessment of automotive components therefore relies on measured load histories, cycle counting techniques and cumulative damage models. Recent investigations confirm that time-domain fatigue analysis combined with rainflow cycle counting remains one of the most reliable approaches for predicting the fatigue life of automotive components subjected to random loading (He, J. et al. 2026; Benasciutti, D., & Marques, J. M. E. 2022).

The materials most frequently investigated in automotive fatigue analysis include high-strength low-alloy steels (HSLA), advanced high-strength steels (AHSS), cast irons, aluminium alloys, magnesium alloys and, in high-performance applications, titanium alloys and fibre-reinforced composites. Typical fatigue-critical components include crankshafts, connecting rods, transmission shafts, suspension control arms, steering knuckles, wheel hubs, brake discs, exhaust manifolds, cylinder heads and welded chassis joints. Since these components operate under highly irregular loading histories, accurate fatigue-life prediction strongly depends on reliable cycle counting procedures capable of representing the actual loading process (He, J. et al. 2026; Benasciutti, D., & Marques, J. M. E. 2022; Bellec, E. et al. 2022).

A major contribution to practical fatigue analysis was made by Wöhler, who experimentally established S–N curves describing the relationship between the stress range S and the number of cycles to failure N (Wöhler, A. 1860). Subsequently, Palmgren introduced the cumulative damage concept (Palmgren, A.G. 1924), which was further developed by Miner into the well-known linear cumulative damage rule that remains one of the fundamental tools for fatigue-life assessment (Miner, M. A. 1945).

Initially, fatigue analysis was limited to constant-amplitude loading. However, real engineering structures are almost exclusively subjected to variable-amplitude loading histories. Consequently, modern fatigue assessment relies on cycle counting methods capable of transforming arbitrary loading histories into an equivalent set of constant-amplitude cycles. These methods are commonly combined with cumulative damage theories and have become indispensable in engineering practice for evaluating fatigue life under realistic operating conditions (Schijve, J. 1963; Dowling, N. E. 1971).

Among the available techniques, the rainflow counting method has become the reference approach because of its ability to identify physically meaningful closed loading cycles corresponding to hysteresis behaviour observed during cyclic deformation. Owing to its accuracy, robustness and compatibility with numerical fatigue analysis procedures, it has been incorporated into numerous engineering standards and commercial fatigue software packages and is widely used in the automotive, aerospace, mechanical and civil engineering industries (He, J. et al. 2026; Benasciutti, D., & Marques, J. M. E. 2022; Benasciutti, D., & Tovo, R. 2023).

The objective of this paper is to present a comparative analysis of the most widely used cycle counting methods for arbitrary load histories. Their theoretical background, computational characteristics, physical interpretation and engineering applicability are discussed and compared through a representative numerical example in order to identify the most suitable method for fatigue assessment under variable-amplitude loading.

METHODS FOR CYCLE COUNTING UNDER ARBITRARY LOAD HISTORIES

If the definition of Wöhler S–N curves represents the most significant milestone in fatigue analysis, the second most important contribution is attributed to Palmgren, A. G. (1924) and Miner, M. A. (1945), whose work led to the formulation of the linear cumulative damage rule, which remains widely used today. Together with cycle counting methods for arbitrary load histories, these approaches form the fundamental tools for practical fatigue analysis.

A wide number of cycle counting methods have been developed, each tailored to specific types of problems. The primary objective of any such method is to transform a variable-amplitude load history into an equivalent set of constant-amplitude cycles. Prior to applying any cycle counting technique, load histories are typically filtered to retain only local extrema, thereby reducing the dataset while preserving essential information.

In general, cycle counting methods can be classified into three main groups:

Peak counting methods focus on identifying peaks within load histories, detecting all local maxima and minima (Mercer, A., & Livesey, J. 1973). This approach decomposes the load history into full cycles; however, it tends to overestimate smaller stress ranges, leading to deviations from actual conditions. Additionally, major cycles may be interrupted when crossing the zero level, causing the largest stress ranges to be neglected. Due to these limitations, peak counting methods are applicable only to specific loading scenarios and have limited general use.

Range counting methods identify half-cycles between relative maxima and minima (Mercer, A., & Livesey, J. 1973). These half-cycles are then paired with corresponding half-cycles of equal magnitude and opposite sign to form full cycles. A key limitation of this approach is the inability to pair all half-cycles due to varying amplitudes. Furthermore, in certain load histories, large-amplitude cycles may be overlooked in favor of smaller ones. Despite these shortcomings, methods within this group—such as the reservoir method and the rainflow counting method—are the most widely used today. When combined with residue processing techniques, they provide highly effective tools for decomposing load histories into full cycles.

Level crossing methods involve defining several load levels (thresholds), and counting the number of times the load signal crosses these levels during loading increments. This method does not provide information on mean stress values or peak magnitudes. Additionally, cycles that do not exceed predefined thresholds are neglected. When the zero level is used as the reference threshold, the method is referred to as the *zero level crossing method*.

The **reservoir method** is typically applied to short time intervals and relatively simple load histories. Due to its simplicity, it is suitable for manual calculations (Knežević, P., & Milanović, M. 2015). This method represents a graphical interpretation of the rainflow method and yields equivalent results (Hoadley, P. W. et al. 2006).

In this approach, the load history is plotted with stress (or another relevant parameter) on the vertical axis and time on the horizontal axis. The considered segment begins and ends at the same maximum value, which also represents the global maximum. The resulting diagram is interpreted as a “reservoir” filled with fluid. The reservoir is then “punctured” at the point corresponding to the global minimum, defining the first cycle through its associated maximum (initial fluid level) and minimum (drain point). The remaining portions form smaller reservoirs, and the procedure is repeated by selecting the next lowest minimum.

Although the reservoir method is a useful engineering tool for fatigue analysis, its application is limited to relatively simple and short load histories. Its key advantages include algorithmic simplicity and the absence of residual (unclosed) cycles, unlike the rainflow method.

The **rainflow counting method (RFC)** is the most widely used technique for analyzing fatigue under variable-amplitude loading. Its acceptance is largely due to its physical interpretation, where each full cycle corresponds to a closed hysteresis loop in the stress–strain diagram.

The concept was first introduced by Matsuishi and Endo (Matsuishi M., & Endo, T. 1968), based on an analogy with raindrops flowing down a pagoda roof. The method identifies half-cycles and pairs them to form full cycles (range counting principle). A flow (half-cycle) is terminated when one of the following conditions is satisfied:

- a flow starting from a local maximum encounters a new flow originating from a higher maximum;
- a flow starting from a local minimum encounters a new flow originating from a lower minimum;
- the flow exits the load history;
- the flow intersects an already initiated flow.

A fundamental limitation of the original algorithm is the requirement for the complete load history, which prevents real-time data processing and demands significant memory storage.

To overcome these limitations, several computational algorithms have been developed, most notably:

- the **three-point criterion**,
- the **four-point criterion**.

These algorithms enable incremental analysis of load histories and extraction of closed hysteresis loops corresponding to full cycles.

The **three-point criterion** considers three successive local extrema P_i, P_{i+1}, i P_{i+2}, and defines stress ranges based on their differences. A cycle is identified and removed from the history when the following condition is satisfied:

$$Y = \left|P_{i+1} - P_i\right| \leq X = \left|P_{i+2} - P_{i+1}\right|, \tag{1}$$

Where *i=1, 2, 3,...,M-2* , and *M* is the length of the data series.

The **four-point criterion** extends this approach by introducing an additional point P_{i+3}, improving robustness in cycle identification. A cycle is extracted when:

$$Z = \left|P_{i+1} - P_i\right| \geq Y = \left|P_{i+2} - P_{i+1}\right| \leq X = \left|P_{i+3} - P_{i+2}\right|. \tag{2}$$

Where *i=1, 2, 3,...,M-3* , and *M* is the length of the data series.

The graphical representation of the three-point and four-point criteria of the rainflow counting method is shown in Figure 1.

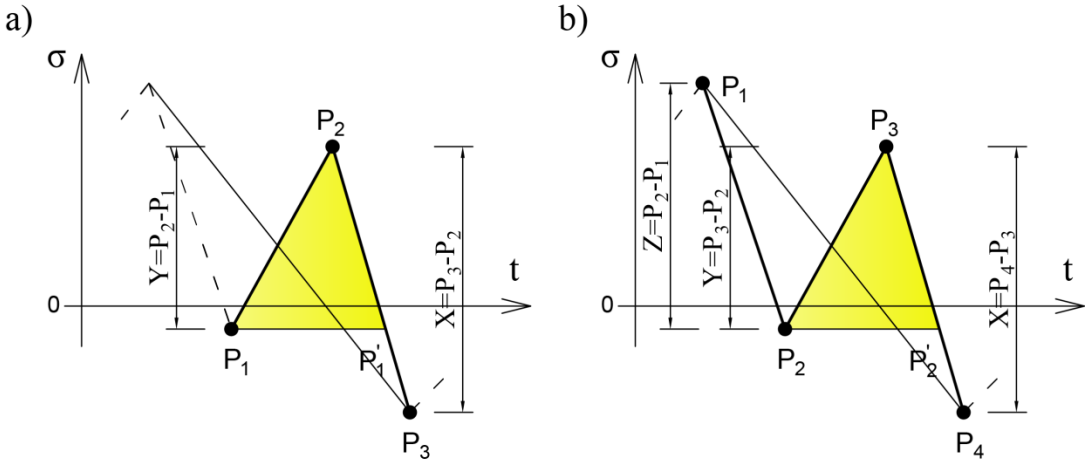


Figure 1 Rainflow Counting Method: a) three-point criterion; b) four-point criterion

These formulations provide a basis for efficient computational algorithms, as presented in (Knežević, P. et al. 2016). Further extensions, such as the **4+1** and **4+n point criteria**, improve computational speed at the expense of increased memory requirements.

Regardless of the applied criterion, residual (unclosed) cycles inevitably remain. Various residue processing techniques have been developed to address this issue (Marsh, G. et al. 2016). It should be noted that different rainflow implementations may yield slightly different results due to variations in algorithm initialization and cycle extraction procedures.

The rainflow method represents a nonlinear numerical algorithm rather than a closed-form mathematical function (Berglind, J. J. B. et al. 2015). Due to its reliance on complete datasets, it is primarily used as a post-processing tool rather than for real-time analysis. Alternative approaches, including stochastic and spectral methods, have been proposed for specific applications involving random loading processes.

NUMERICAL EXAMPLE AND COMPARISON

In order to illustrate the differences between various cycle counting methods and to evaluate their applicability in fatigue analysis, a representative numerical example is considered. The selected load history (Figure) corresponds to a sequence of stress extrema, previously filtered to retain only local maxima and minima, which is a standard preprocessing step in fatigue analysis.

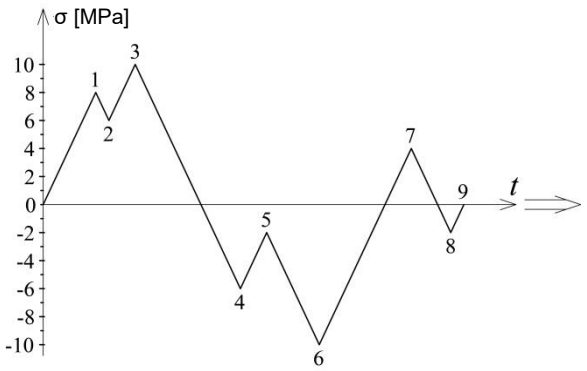


Figure 2 Load history

The primary objective of this example is to demonstrate how different cycle counting methods interpret the same variable-amplitude load history and how this affects the resulting cycle distribution. Since fatigue damage assessment is highly sensitive to the identified stress ranges and the number of cycles, even small differences between methods may lead to significant discrepancies in predicted fatigue life.

The considered load history is represented by stress values at its characteristic points in Table 1.

Table 1 Load history numerical data

Point	1	2	3	4	5	6	7	8	9
σ [MPa]	8	6	10	-6	-2	-10	4	-2	0

The analysis includes the application of the following groups of methods:

- peak counting methods,
- range counting methods (including the reservoir method and the rainflow counting method),
- level crossing methods.

For each method, the cycle counting procedure is presented step by step in order to clearly demonstrate how individual cycles are identified and extracted from the load history. Special attention is given to the treatment of large-amplitude cycles, the handling of residuals, and the pairing of half-cycles.

The results obtained using each method are presented in tabular form, including:

- stress range Δσ,
- mean stress (where applicable),
- number of cycles.

Finally, a comparative analysis is performed to highlight the key differences between the methods in terms of:

- accuracy and physical relevance,
- sensitivity to load sequence effects,
- computational efficiency and practical applicability.

This comparison provides a basis for selecting the most appropriate cycle counting method in engineering practice, particularly in the context of fatigue analysis of structures subjected to variable-amplitude loading.

Summary of Cycle Counting Results

The results of cycle counting for the given load history using different methods are summarized in the following tables.

Table 2 Results of cycle counting using different methods

Peak Counting Method

Stress Range Δσ	Number of Cycles
2	1
4	1
6	1
8	1
10	1
11	1

Table 2 (continuation)

Range Counting Method

Stress Range Δσ	Number of Cycles
2	1
4	1
6	1
8	1
14	1

Reservoir Method

Stress Range Δσ	Number of Cycles
20	1
16	1
10	1
6	1

Rainflow Counting Method (Reference)

Stress Range Δσ	Number of Cycles
20	1
14	1
10	1
6	1
4	1
2	1

Level Crossing Method

Stress Range Δσ	Number of Cycles
—	Not directly defined
Crossings (0-level)	6

Table 3 Comparative Overview of cycle counting methods

Method	Captures Large Cycles	Small Cycle Sensitivity	Residue	Practical Use
Peak counting	No	High (overestimation)	No	Limited
Range counting	Partial	Medium	Yes	Moderate
Reservoir	Yes	Medium	No	Manual analysis
Rainflow	Yes (accurate)	Balanced	Yes	Standard method
Level crossing	No	Low	—	Statistical use

CONCLUSIONS

The comparative analysis of cycle counting methods highlights significant differences in their ability to represent variable amplitude loading histories. Peak and level crossing methods show limited applicability due to their inability to accurately capture dominant stress cycles. Range-based methods provide improved results but still exhibit limitations in handling complex load sequences. The reservoir method offers a simple and intuitive approach suitable for manual calculations. However, the rainflow counting method proves to be the most accurate and widely applicable technique, as it provides a physically consistent representation of fatigue cycles and has become the most commonly adopted approach for fatigue life assessment in engineering practice.

Nevertheless, despite its advantages, the rainflow method also exhibits certain limitations, particularly related to the simplification of complex loading histories into equivalent closed cycles and the neglect of detailed material response during cyclic loading. Future research should therefore focus on the development of advanced cycle identification approaches based on models that consider the physical nature of complete fatigue cycles, such as various hysteresis-based models. Such approaches could provide a more accurate description of material behavior under arbitrary loading conditions by incorporating load sequence effects, cyclic plasticity, and the evolution of hysteretic energy dissipation. This could lead to improved fatigue life prediction methods beyond the capabilities of conventional cycle counting techniques.

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